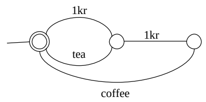
CCS: Processes and Equivalences

Reading: Peled 8.5

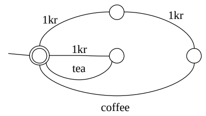
Mads Dam

Finite State Automata

- Coffee machine A_1:



- Coffee machine A_2:



- Are the two machines "the same"?

Little Language Refresher

Automata recognize strings, e.g.

1kr tea 1kr 1kr coffee

Language = set of strings

A_1 "=" language recognized by A_1

ε: empty string

ST: concatenation of *S* and *T*

S + *T*: union of *S* and *T*

*S**: Iteration of *S*

$S^* = \underbrace{SS...S}_{\text{0 or more times}}$

Arden's rule:

The equation

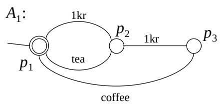
$X = SX + T$

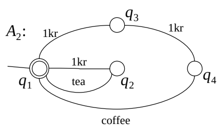
has solution

$X = S^*T$

If $\varepsilon \notin S$ the solution is unique

Now compute...





For A_1:

$p_1 = 1kr\ p_2 + \varepsilon$

$p_2 = 1kr\ p_3 + tea\ p_1$

$p_3 = coffee\ p_1$

$\Downarrow$

$p_2 = 1kr\ coffee\ p_1 + tea\ p_1$

$\Downarrow$ (etc, use Arden)

$p_1 = (1kr\ 1kr\ coffee + 1kr\ tea)^*$

$= q_1$

BUT: p_1, q_1 should be different!

SO: need new theory to talk about *behaviour* instead of *acceptance*

Process Algebra

Calculus of concurrent processes

Main issues:

- How to specify concurrent processes in an abstract way?
- Which are the basic relations between concurrency and non-determinism?
- Which basic methods of construction (= operators) are needed?
- When do two processes behave differently?
- When do they behave the same?
- Rules of calculation:
  - Replacing equals for equals
  - Substitutivity
- Specification and modelling issues

Process Equivalences

Sameness of behaviour = equivalence of states

Many process equivalences have been proposed (cf. Peled 8.5)

For instance: $q_1 \sim q_2$ iff

- q_1 and q_2 have the same paths, *or*
- q_1 and q_2 may always refuse the same interactions, *or*
- q_1 and q_2 pass the same tests, *or*
- q_1 and q_2 have identical branching structure

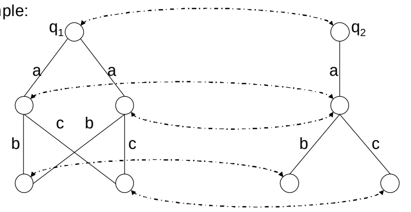
CCS: Focus on <u>bisimulation equivalence</u>

Bisimulation Equivalence

Intuition: $q_1 \sim q_2$ iff q_1 and q_2 have same branching structure

Idea: Find relation which will relate two states with the same transition structure, and make sure the relation is preserved

Example:



Strong Bisimulation Equivalence

Given: Labelled transition system $T = (Q,\Sigma,R)$

Looking for a relation $S \subseteq Q \times Q$ on states

S is a *strong bisimulation relation* if whenever q_1 S q_2 then:

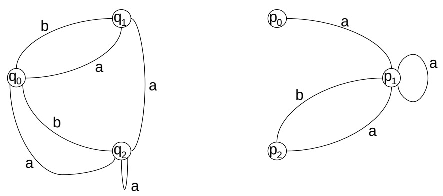
- $q_1 \to^\alpha q_1'$ implies $q_2 \to^\alpha q_2'$ for some q_2' such that q_1' S q_2'
- $q_2 \to^\alpha q_2'$ implies $q_1 \to^\alpha q_1'$ for some q_1' such that q_1' S q_2'

q_1 and q_2 are *strongly bisimilar* iff q_1 S q_2 for some strong bisimulation relation S

$q_1 \sim q_2$: q_1 and q_2 are strongly bisimilar

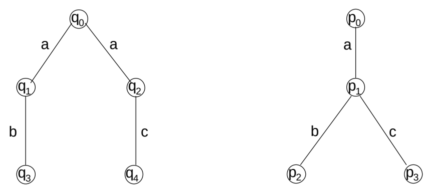
Peled uses $\equiv_{bis}$ for $\sim$

Example



Does $q_0 \sim p_0$ hold?

Example



Does $q_0 \sim p_0$ hold?

Weak Transitions

What to do about internal activity?

τ: Transition label for activity which is not externally visible

- $q \Rightarrow^\varepsilon q'$ iff $q = q_0 \to^\tau q_1 \to^\tau \ldots \to^\tau q_n = q'$, $n \geq 0$
- $q \Rightarrow^\tau q'$ iff $q \Rightarrow^\varepsilon q'$
- $q \Rightarrow^\alpha q'$ iff $q \Rightarrow^\varepsilon q_1 \to^\alpha q_2 \Rightarrow^\varepsilon q'$ $(\alpha \neq \tau)$

Beware that $\Rightarrow^\tau = \Rightarrow^\varepsilon$ (non-standard notation)

Observational equivalence, v.1.0: Bisimulation equivalence with $\Rightarrow$ in place of $\to$

Let $q_1 \approx' q_2$ iff $q_1 \sim q_2$ with $\Rightarrow^\alpha$ in place of $\to^\alpha$

Cumbersome definition: Too many transitions $q \Rightarrow^\alpha q'$ to check

Observational Equivalence

Let $S \subseteq Q \times Q$. The relation S is a *weak bisimulation relation* if whenever q_1 S q_2 then:

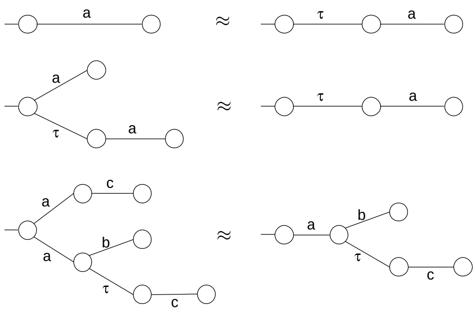
- $q_1 \to^\alpha q_1'$ implies $q_2 \Rightarrow^\alpha q_2'$ for some q_2' such that q_1' S q_2'
- $q_2 \to^\alpha q_2'$ implies $q_1 \Rightarrow^\alpha q_1'$ for some q_1' such that q_1' S q_2'

q_1 and q_2 are *observationally equivalent,* or *weakly bisimulation equivalent,* if q_1 S q_2 for some weak bisimulation relation S

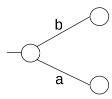
$q_1 \approx q_2$: q_1 and q_2 are observationally equivalent/weakly bisimilar

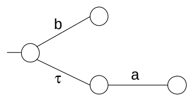
Exercise: Show that $\approx' = \approx$

Examples



Examples





All three are inequivalent