2G1516 Formal Methods – HT05
CTL Exercises

Exercise 1 Two CTL formulas, ϕ and ψ, are equivalent, $\phi \equiv \psi$, if for all transition systems T and all interpretation functions ρ, $T \models \phi$ iff $T \models \psi$ (using ρ). Prove, using the semantics of CTL, the equivalence:

$$A(\phi \mathsf{U} \psi) \equiv \neg(E(\neg\psi \mathsf{U} (\neg\phi \wedge \neg\psi)) \vee EG\neg\psi)$$

□

Exercise 2 For each state in the model given in Figure 1 determine whether or not the following CTL formulas hold for that state:

1. EGp
2. AGp
3. $EFAGp$
4. $A(p \mathsf{U} EG(p \rightarrow q))$
5. $E(((p \wedge q) \vee r) \mathsf{U} (E(r \mathsf{U} AGp)))$

□

Definition: We use $|\phi|$ to denote the *length* of formula ϕ, that is, the number of symbols in ϕ viewed as a string over the set of atomic propositions union the set of connectives ($\wedge$, $\neg$, A, G, F, U, (,), etc.).

Exercise 3 (advanced)

Construct models M_i and N_i, as in Figure 2. Prove for all CTL formulas ϕ, that whenever $|\phi| \leq i$ then

1. $s_i \models \phi$ iff $s'_i \models \phi$
2. $t_i \models \phi$ iff $t'_i \models \phi$

Conclude that the LTL property $<> []p$ (almost always p) is not expressible in CTL.

Hint: Prove first, by induction on $|\phi|$, that for any CTL formula ϕ,

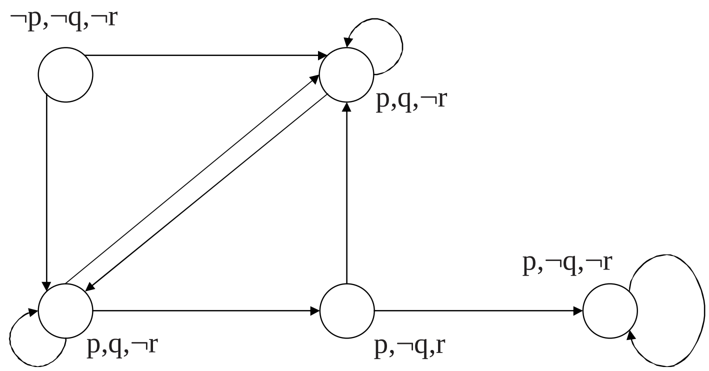

Figure 1: Model for Exercise 2.

- $(\exists j \geq |\phi|.\ s'_j \models \phi)$ iff $(\forall j \geq |\phi|.\ s'_j \models \phi)$
- $(\exists j \geq |\phi|.\ t'_j \models \phi)$ iff $(\forall j \geq |\phi|.\ t'_j \models \phi)$.

$\square$

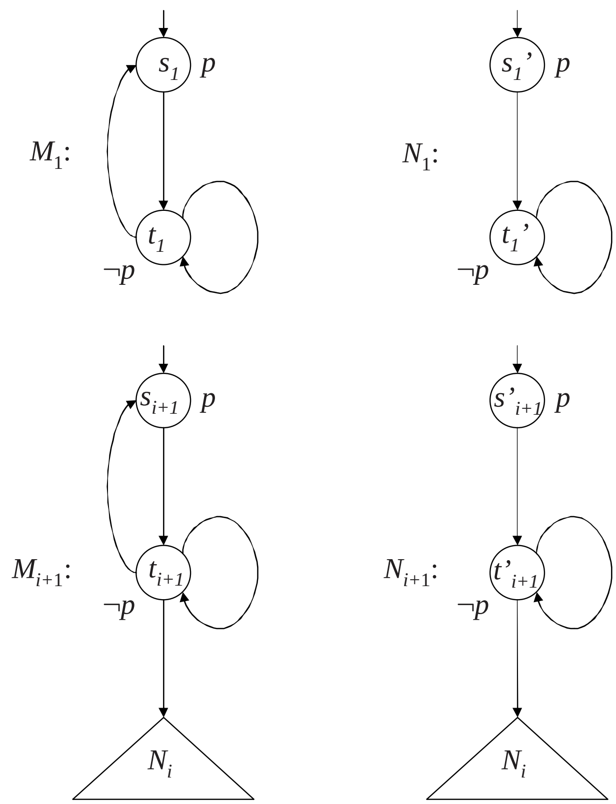

Figure 2: Model for Exercise 3.