2G1516 Formal Methods – HT05
CCS - I

Exercise 1 List all members of the set $\{P' \mid ((\alpha.P \mid \beta.Q) \mid (\overline{\alpha}.R + \overline{\beta}.S)) \xrightarrow{\tau} P'$.
$\square$

Exercise 2 Consider the following CCS process definitions:

$$\begin{array}{lcl} Spec & \stackrel{\text{def}}{=} & in.\overline{out}.Spec \\ Sender & \stackrel{\text{def}}{=} & in.Transmit \\ Transmit & \stackrel{\text{def}}{=} & \overline{transmit}.WaitAck \\ WaitAck & \stackrel{\text{def}}{=} & ack_+.Sender + ack_-.Transmit \\ Receiver & \stackrel{\text{def}}{=} & transmit.Analyze \\ Analyze & \stackrel{\text{def}}{=} & \tau.\overline{out}.\overline{ack_+}.Receiver + \tau.\overline{ack_-}.Receiver \\ Protocol & \stackrel{\text{def}}{=} & (Sender|Receiver)\backslash\{transmit, ack_+, ack_-\} \end{array}$$

Derive the underlying transition graphs. $\square$

Exercise 3 Modify the definition of the two-way buffer in the slides so that it functions reliably. $\square$

Exercise 4 Suppose that R, R_1 and R_2 are strong bisimulation relations. Are the following relations also strong bisimulation relations?

1. R^{-1}
2. $R_1 \cup R_2$
3. $R_1 \cap R_2$
4. $R_1 \circ R_2$ ($\circ$ is relational composition)

$\square$

Exercise 5 The transition system $(Q, \Sigma, \rightarrow)$ is *image-finite*, if for any P and α, $\{P' \mid P \xrightarrow{\alpha} P'$ is a finite set. A set of CCS definitions $\{A_i == P_i \mid 1 \leq i \leq n\}$ is *guarded*, if all occurrences of any A_i in any P_j, $1 \leq i, j, \leq n$, is within the scope of a prefix (so that e.g. $X == \alpha.X$ is guarded, but $X == X + (X \mid X)$ is not). A CCS term P is guarded if all its accompanying definitions are. Prove that if P is guarded then its corresponding transition system is image-finite. $\square$

Exercise* 6 Let $T = (Q, \Sigma, \rightarrow)$ be a labelled transition system. Define the function F on relations $R \subseteq Q \times Q$ by $F(R)(q_1, q_2)$ iff:

- For all $q_1' \in Q$, if $q_1 \xrightarrow{\alpha} q_1'$ then there is some $q_2' \in Q$ such that $q_2 \xrightarrow{\alpha} q_2'$ and $q_1' R q_2'$.
- For all $q_2' \in Q$, if $q_2 \xrightarrow{\alpha} q_2'$ then there is some $q_1' \in Q$ such that $q_1 \xrightarrow{\alpha} q_1'$ and $q_1' R q_2'$.

Define the family of relations $\sim_n \subseteq Q \times Q$, for n a natural number, inductively in the following way:

1. $\sim_0 = Q \times Q$
2. $\sim_{n+1} = F(\sim_n)$

Prove:

1. $\sim \subseteq \sim_n$ for all n
2. $\sim = \bigcup_n \sim_n$ if $\xrightarrow{\alpha}$ is image-finite.

□

Exercise 7 Prove that $\sim$ is an equivalence relation. □

Exercise 8 A *simulation* is a relation R such that whenever $q_1 R q_2$ and $q_1 \xrightarrow{\alpha} q_1'$ then some q_2' exists such that $q_2 \xrightarrow{\alpha} q_2'$ and $q_1' R q_2'$, but not necessarily conversely. Show that it is possible for both R and R^{-1} to be simulations, without R being a bisimulation. □

Exercise 9 Determine whether or not the following observational equivalences hold:

1. $b.0 + \tau.(a.0 + \tau.b.0) \approx b.0 + \tau.(a.0 + b.0)$
2. $b.0 + \tau.(a.0 + \tau.b.0) \approx b.0 + \tau.b.0 + \tau.(a.0 + \tau.b.0)$

If you claim the equivalences hold, establish a weak bisimulation relation. If you claim they do not, explain why. □

Exercise 10 Determine whether or not the following observational equivalences hold:

1. $a.0 \mid b.0 \approx a.0 \mid \tau.b.0$
2. $a.0 \mid b.0 \approx \tau.a.0 \mid \tau.b.0$

If you claim the equivalences hold, establish a weak bisimulation relation. If you claim they do not, prove that no such relation can exist. □