2G1516 Formal Methods – HT05
Buchi Automata Exercises

Exercise 1 Construct Büchi automata accepting the following ω-languages over $\Sigma = \{a, b, c\}$.

1. $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains at least one infix $ab\}$
2. $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains infix ab infinitely often$\}$
3. $L_3 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains infix ab only finitely often$\}$
4. $L_4 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains at least one infix ab and at least one infix $ac\}$
5. $L_5 = \{\alpha \in \Sigma^\omega \mid$ if α contains infinitely many a's then α contains infinitely many b's$\}$

Hint: For 5, first rephrase the condition on α as a disjunction of cases. □

Exercise 2 Construct deterministic Büchi automata accepting the following ω-languages over $\Sigma = \{a, b, c\}$.

1. $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains at least one letter $c\}$
2. $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha$ does not contain a letter $c\}$
3. $L_3 = \{\alpha \in \Sigma^\omega \mid$ in α, every a is immediately followed by $b\}$
4. $L_4 = \{\alpha \in \Sigma^\omega \mid$ in α, in between two successive a's there are at least two b's$\}$

□

Exercise 3 For each LTL formula f_i below, using the semantics of LTL create a Büchi automaton $\mathcal{A}_i$ which accepts the language $\{w \in (\Sigma_i)^w \mid w \models f_i\}$, where $\Sigma_i = 2^{\mathrm{AP}_i}$. (The language contains exactly those infinite words which are models of the formula.)

1. $\mathrm{AP}_1 = \{p\}$, $f_1 = \Box\Diamond p$
2. $\mathrm{AP}_2 = \{p\}$, $f_2 = \Diamond\Box\neg p$
3. $\mathrm{AP}_3 = \{p, q\}$, $f_3 = p \mathsf{U} q$

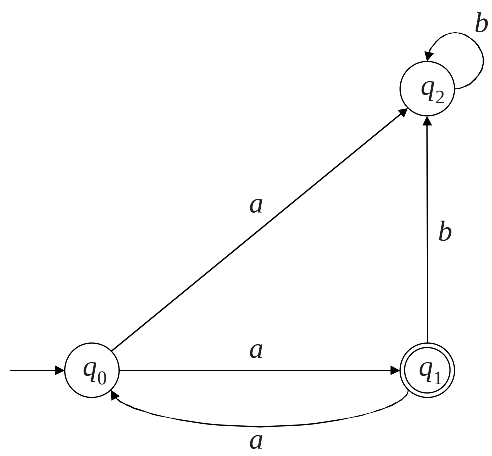

Figure 1: $\mathcal{A}_1$

4. $\mathrm{AP}_4 = \{p, q\}$, $f_4 = (\Diamond\Box p) \rightarrow (\Diamond\Box q)$
5. $\mathrm{AP}_5 = \{p\}$, $f_5 = \bigcirc \bigcirc p$

$\Box$

Exercise 4 Given $\Sigma = \{a, b\}$, consider the Büchi automata $\mathcal{A}_1$ and $\mathcal{A}_2$ on figures 1 and 2 respectively.

1. Is it true that $\mathcal{L}(\mathcal{A}_1) = \emptyset$?
2. Does the automaton $\mathcal{A}_1$ accept the infinite word $(a)^w$?
3. Does the automaton $\mathcal{A}_1$ accept the infinite word $a(b)^w$?
4. Does the automaton $\mathcal{A}_2$ accept $(abb)^w$. If it does, give an accepting run of the automaton.
5. Construct a Büchi automaton $\mathcal{A}_e$ that recognizes the language $\mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\mathcal{A}_2)$.
6. Is it true that $\mathcal{L}(\mathcal{A}_e) = \emptyset$? If not, give an accepting run of the automaton $\mathcal{A}_e$.

$\Box$

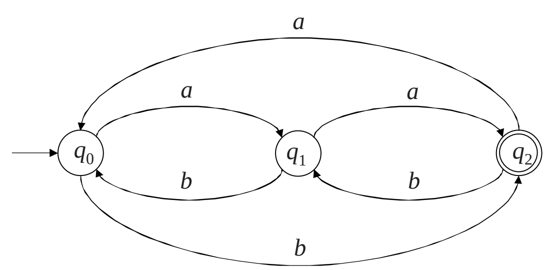

Figure 2: $\mathcal{A}_2$