2G1516 Formal Methods   2G1521 Formal Methods for SEDS

EXAMINATION PROBLEMS — *Mads Dam*
10 January 2005, 3pm–8pm — KTH/IMIT

Give solutions in English or Swedish, each problem beginning on a new sheet. Write your name on all sheets. The maximal number of points is given for each problem. Textbook, copies of slides, other written course material and English dictionaries are admissible. Computers, mobile phones, other written material, is not.
Grades are given in the range F, 3, 4, 5 with the following cut-off points: 3: 45, 4: 60, 5: 80

---

1. Prove each of the following formulas using the basic PVS proof rules. **15p**

   1. $\exists x : \forall y : P(x, y) \rightarrow \forall y : \exists x : P(x, y)$
   2. $(\forall x : P(x) \rightarrow \exists y : \forall z : Q(y, z)) \rightarrow ((\exists x : P(x)) \rightarrow \exists y : \forall z : Q(y, z))$

---

2. In the slides, the semantics of LTL is given in terms of a relation $\xi \models \phi$ where ξ is an execution sequence and ϕ is an LTL formula. Reformulate this semantics as a relation $\xi, i \models \phi$ with the intention that ϕ holds at the i'th point in ξ, and for a transition system T let $T \models \phi$ iff for all runs ξ of T and for all i, $\xi, i \models \phi$. **15p**
   Using your modified semantics introduce new connectives S ("since") and P ("previously") with the informal meanings that:

   - $\phi S \psi$ holds at point i in the execution sequence ξ if ψ held some time in the past and since then ϕ has held.
   - $P\phi$ holds at point i in the execution sequence ξ if $i \geq 1$ and ϕ holds at point $i - 1$.

   Express, using only S and propositional connectives the properties:

   1. Some time in the past, ϕ has held.
   2. Always in the past, ϕ has held.
   3. ϕ has held exactly once in the past, the present not included.

---

3. Construct a Buchi automaton accepting the language $aU([]b)$ **15p**

Please Turn Over

4. Produce a transition diagram to compute the factorial $y = x!$ from input $x \geq 1$. Show that the transition diagram is partially correct with respect to the appropriate pre- and postcondition. 25p

5. Determine whether or not the following observational equivalences hold: 15p

   1. $b.0 + \tau.(a.0 + \tau.b.0) \approx b.0 + \tau.(a.0 + b.0)$
   2. $b.0 + \tau.(a.0 + \tau.b.0) \approx b.0 + \tau.b.0 + \tau.(a.0 + \tau.b.0)$

   If you claim the equivalences hold, establish a weak bisimulation relation. If you claim they do not, explain why (you do not have to give a formal proof).

6. A 2-bit counter is specified in CCS as a process C^2: 15p

   - $C^2 == C_0^2$
   - $C_0^2 == \overline{zero}.C_0^2 + inc.C_1^2$
   - $C_1^2 == \overline{nonzero}.C_1^2 + inc.C_0^2$

   Show how a 4-bit counter can be implemented in CCS by two 2-bit counters. That is, specify a 4-bit counter as a process C^4. Specify also a control process P which will use two copies of C^2 to implement C^4 as a process

   $$Q == (P|C^2[f_1]|C^2[f_2]\ L$$

   where f_1 and f_2 are suitable relabelling functions, and L is the set of internal names through which P and the 2-bit counters communicate. You are not required to prove that $C^4 \approx Q$.

Good luck!