2G1516 Formal Methods, 2G1521 Formal Methods for SEDS

Examination Problems — *Mads Dam*
18 October 2004, 3pm–8pm — KTH/IMIT

Give solutions in English or Swedish, each problem beginning on a new sheet. Write your name on all sheets. The maximal number of points is given for each problem. Textbook, copies of slides, other written course material and English dictionaries are admissible. Computers, mobile phones, other computing or communication equipment, is not.
Grades are given in the range F, 3, 4, 5 with the following cut-off points: 3: 45, 4: 60, 5: 80

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1. **[15p]**

   1. flatten; skolem 1 y; skolem -1 x; flatten; inst -2 y; inst 1 x; split
   2. Let $P(x)$ be *true* and $Q(x,y)$ be $x = y$. Then $\forall x : P(x) \to \exists y : Q(x,y)$ is valid, but $\exists y : \forall x : P(x) \to Q(x,y)$ is not when the domain of x and y has more than 1 element.

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2. Atomic propositions are en_e (event e is enabled) and ex_e (event e is executing/takes place), where $e \in \{i, l, w, j\}$. **[15p]**

   1. $[]((ex_i \to en_i) \wedge (ex_l \to en_l) \wedge \cdots \text{etc.})$
   2. $[] <> ex_i \to [] <> ex_j$
   3. $[](en_l \leftrightarrow en_w)$
   4. $[]((ex_i \vee ex_l \vee ex_w \vee ex_j) \wedge (ex_i \to \neg(ex_l \vee ex_w \vee ex_j)) \wedge (ex_l \to \neg(ex_i \vee ex_w \vee ex_j)) \wedge \cdots \text{etc.})$
   5. $[] <> (en_j \wedge \neg ex_j)$
   6. $[](ex_j \to O((\neg en_w) U ex_i))$

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3. See figure 1. **[15p]**

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4. Variables x, y, z range over integers. pc, pc_1, pc_2 range over $\{1, 2, 3\}$. States are vectors $< pc, pc_1, pc_2, x, y, z >$. The initial condition is $pc = 1$ (pc for main program has initial value). The transition specifications are: **15p**

- $pc = 1 \to (pc, pc_1, pc_2, z) := (2, 1, 1, 0)$
- $pc = 2 \wedge pc_1 = 1 \wedge x \neq 0 \to (pc_1, z) := (2, z + 1)$
- $pc = 2 \wedge pc_1 = 2 \to (pc_1, x) := (1, x - 1)$
- $pc = 2 \wedge pc_1 = 1 \wedge x = 0 \to pc_1 := 3$
- $pc = 2 \wedge pc_2 = 1 \wedge y \neq 0 \to (pc_2, z) := (2, z + 1)$
- $pc = 2 \wedge pc_2 = 2 \to (pc_2, y) := (1, y - 1)$
- $pc = 2 \wedge pc_2 = 1 \wedge y = 0 \to pc_2 := 3$
- $pc = 2 \wedge pc_1 = 3 \wedge pc_2 = 3 \to pc = 3$

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5. **20p**

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{x = x0 && x0 >= 0 && z = 0}
{x0 = x + z && x >= 0}
while x != 0 do
  {x0 = x + z && x >= 0 && x != 0}
  z := z + 1 ;
  {x0 + 1 = x + z && x >= 0 && x != 0}
  x := x - 1
  {x0 = x + z && x >= 0}
od
{x0 = x + z && x >= 0 && x = 0}
{z = x0}
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6. **20p**

1. Let $P = a.0|b.0$, $P_a = 0 \mid b.0$, $Q = a.0 \mid \tau.b.0$, $Q_a = 0 \mid \tau.b.0$, and let Id be the identity relation on CCS terms. The relation $S_1 = Id \cup \{(P, Q), (P_a, Q_a)\}$ is a weak bisimulation relation.
2. Let $P = a.0|b.0$, $P_a = 0 \mid b.0$, $P_b = a.0 \mid 0$, $Q = \tau.a.0 \mid \tau.b.0$, $Q_{\tau,1} = a.0 \mid \tau.b.0$, $Q_{\tau,2} = \tau.a.0 \mid b.0$, $Q_a = 0 \mid \tau.b.0$ and $Q_b = \tau.a.0 \mid 0$. The relation $S_2 = Id \cup \{(P, Q), (P, Q_{\tau,1}), (P, Q_{\tau,2}), (P_a, Q_a), (P_b, Q_b)\}$ is a weak bisimulation relation.

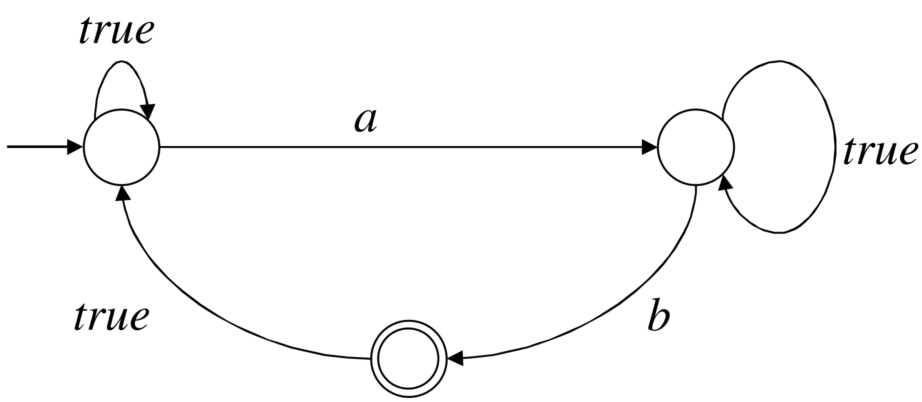

Figure 1: Buchi automaton for exercise 3