

2G1516 Formal Methods

EXAMINATION PROBLEMS
18 December 2003, 8am–1pm

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Give solutions in English or Swedish, each problem beginning on a new sheet. Write your name on all sheets. The maximal number of points is given for each problem. Textbook, copies of slides, other written course material and English dictionaries are admissible. Computers, mobile phones, other written material, is not. Grades are given in the range F, 3, 4, 5 with the following cut-off points: 3: 45, 4: 70, 5: 90

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1. Let ϕ_0, ϕ_1, ϕ_2 be formulas in LTL. The property $Cyclic(\phi_0, \phi_1, \phi_2)$ holds of a given path ξ iff 12p
1. for all $k \geq 0$, $\xi^k \models \phi_i$ for some $i \in \{0, 1, 2\}$,
 2. for all $k \geq 0$ and $i \in \{0, 1, 2\}$, if $\xi^k \models \phi_i$ then $\xi^{k+1} \models \phi_i$ or $\xi^{k+1} \models \phi_{i \oplus 1}$, and
 3. for each $i \in \{0, 1, 2\}$ there are infinitely many $k \geq 0$ such that $\xi^k \models \phi_i$.
- (\oplus is addition modulo 3). Express $Cyclic(\phi_0, \phi_1, \phi_2)$ in LTL.
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2. Let $A_i = (Q_i, \Sigma, \Delta_i, I_i, F_i)$, $i \in \{1, 2\}$, be two arbitrary Buchi automata such that $F_1 = Q_1$. Give a *direct* construction of a Buchi automaton $A_1 \otimes A_2$ such that $L(A_1 \otimes A_2) = L(A_1) \cap L(A_2)$. Your solution must avoid using the generalized Buchi automata construction of the slides/Peled. 12p
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3. Two sequential programs c_1, c_2 are *pre-post-equivalent*, $c_1 \simeq c_2$, if for all FOL formulas ϕ, ψ , $\{\phi\}c_1\{\psi\}$ iff $\{\phi\}c_2\{\psi\}$. Is \simeq preserved by parallel composition, i.e. is it the case that $c_1 \simeq c_2$ implies $c \parallel c_1 \simeq c \parallel c_2$ and $c_1 \parallel c \simeq c_2 \parallel c$? If your answer is yes, give a proof. If your answer is no, give a counterexample and explain. 16p
- (Note: In the slides $c \parallel c'$ is also written $cobegin\ c \parallel\ c' \ coend$. These forms are synonymous.)
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4. Let c be the following command: 18p
- $$\begin{aligned} &(done1, done2) := (false, false) ; \\ &cobegin\ (x, done1) := (x+1, true) \parallel (x, done2) := (x+1, true) \ coend \end{aligned}$$
- Use Owicki-Gries to prove $\{x = 0\}c\{x = 2\}$.
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Please Turn Over

5. Let P and Q be CCS processes such that $P \xrightarrow{\tau} Q$ and $Q \xrightarrow{\tau} P$ (P and Q may have other transitions, but they have at least these transitions). Prove that $P = Q$. 12p

6. Let

$$\begin{aligned}
 P &== b.c.0 + b.d.0 \\
 Q &== P + b.(c.0 + d.0) \\
 R &== a.Q + a.P \\
 S &== a.Q
 \end{aligned}$$

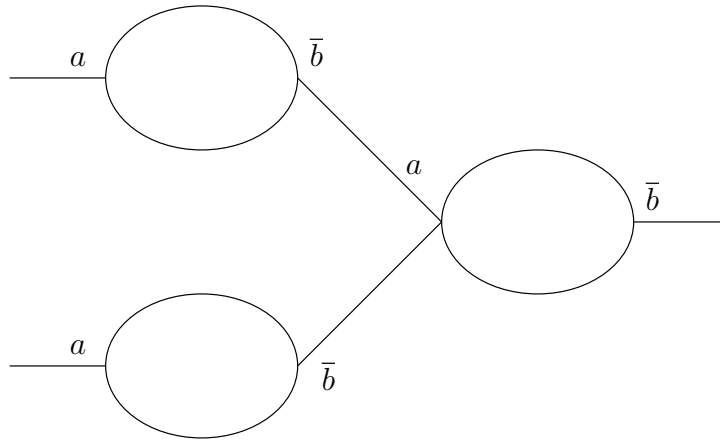
12p

Give a formula in HML which is true for R and false for S .

7. An n -trigger, $TR(n)$, is a process with two ports, a and \bar{b} defined inductively as follows: 18p

- $TR(0) == \bar{b}.0$
- $TR(n+1) == a.TR(n)$

Consider the following system S of three 2-triggers:



Define this system using three copies of $TR(2)$, parallel composition, relabelling, and restriction. Draw the transition graphs of the system. Determine that S is weakly bisimilar to $TR(4)$.

Good luck!